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1. (1 pt) Use Stoke's theorem to evaluate $\int\int_S curl\mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = -9yz\mathbf{i} + 9xz\mathbf{j} + 5(x^2+y^2)z\mathbf{k}$ and S is the part of the paraboloid $z = x^2+y^2$ that lies inside the cylinder $x^2+y^2 = 1$, oriented upward.

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2. (1 pt) Use Stoke's Theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + 3(x^2+y^2)\mathbf{k}$ and C is the boundary of the part of the paraboloid where $z = 9 - x^2 - y^2$ which lies above the xy-plane and C is oriented counterclockwise when viewed from above.

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3. (1 pt) Use the divergence theorem to find the outward flux of the vector field $\mathbf{F}(x,y,z) = 1x^2\mathbf{i} + 4y^2\mathbf{j} + 4z^2\mathbf{k}$ across the boundary of the rectangular prism: $0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 1$.

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4. (1 pt) Let $\mathbf{F} = (2x, 2y, 2x+2z)$.

Use Stokes' theorem to evaluate the integral of $\mathbf{F}$ around the triangle C consisting of the straight lines joining the points (1,0,1), (0,1,0) and (0,0,1).

Let S be the triangular surface whose boundary is the three line segments.

Compute $\mathbf{n}$, the unit normal vector of S, and the curl of $\mathbf{F}$.

$\mathbf{n} =$ (_____,_____,_____) (the unit normal vector)

$\nabla\times\mathbf{F} =$ (_____,_____,_____)

Using Stokes' theorem, the value of the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ is

$\int_C \mathbf{F}\cdot d\mathbf{r} = \int\int_S \nabla\times\mathbf{F}\cdot d\mathbf{S} = \int\int_S \nabla\times\mathbf{F}\cdot\mathbf{n}\, dS =$ _________.

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5. (1 pt) Gauss's law in integral form states that the charge Q enclosed by a surface S is a constant (ε_0) times the flux of the electric field $\mathbf{E}$ outward through S. (The orientation of the surface S is outward.)

$$Q = \varepsilon_0 \int\int_S \mathbf{E}\cdot d\mathbf{S}$$

Find the charge enclosed by the cube with vertices $(\pm3, \pm3, \pm3)$ if the electric field is $\mathbf{E}(x,y,z) = -x\mathbf{i} - y\mathbf{j} - 2z\mathbf{k}$.

$Q =$ __________________ ε_0

The easiest way to do this problem is to apply the divergence theorem. When you do, you will rediscover the differential form of Gauss's law: The charge density ρ is given by $\rho = \varepsilon_0\nabla\cdot\mathbf{E}$.

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6. (1 pt)

Calculate the surface integral $\int\int_M (\nabla\times\mathbf{F})\cdot d\mathbf{S}$ where M is the hemisphere $x^2+y^2+z^2 = 4, x \ge 0$, with the normal in the direction of the positive x direction, and $\mathbf{F} = (x^4, 0, y)$.

Begin by writing down the "standard" parametrization of ∂M as a function of t.

$x =$ _________, $y =$ _________, $z =$ _________.

$\int\int_M (\nabla\times\mathbf{F})\cdot d\mathbf{S} = \int_{\partial M}\mathbf{F}\cdot d\mathbf{r} = \int_0^{2\pi} f(t)\, dt$, where

$f(t) =$ _______________________.

The value of the integral is ________.

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7. (1 pt)

Verify Stokes' theorem for the helicoid $\Psi(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$ where (r,θ) lies in the rectangle $[0,1]\times[0,\pi/2]$, and $\mathbf{F}$ is the vector field $\mathbf{F} = (8z, 5x, 3y)$.

First, compute the surface integral:

$\int\int_M (\nabla\times\mathbf{F})\cdot d\mathbf{S} = \int_a^b\int_c^d f(r,\theta)\,dr\,d\theta$, where

$a =$____, $b =$____, $c =$____, $d =$____, and

$f(r,\theta) =$ _______________________ (use "t" for theta).

Finally, the value of the surface integral is ____________.

Next compute the line integral on that part of the boundary from $(1,0,0)$ to $(0,1,\pi/2)$.

$\int_C \mathbf{F}\cdot d\mathbf{s} = \int_a^b g(\theta)\, d\theta$, where

$a =$____, $b =$____, and

$g(\theta) =$ _______________________ (use "t" for theta).

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8. (1 pt)

Let M be the capped cylindrical surface which is the union of two surfaces, a cylinder given by $x^2+y^2 = 36,\ 0 \le z \le 1$, and a hemispherical cap defined by $x^2+y^2+(z-1)^2 = 36,\ z \ge 1$. For the vector field $\mathbf{F} = (zx + z^2y + 5y,\ z^3yx + 3x,\ z^4x^2)$, compute $\int\int_M (\nabla\times\mathbf{F})\cdot d\mathbf{S}$ in any way you like.

$\int\int_M (\nabla\times\mathbf{F})\cdot d\mathbf{S} =$ ____________

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9. (1 pt)

Evaluate $\int\int_{\partial W}\mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = (x^2+y, z^2, e^y - z)$ and W is the solid rectangular box whose sides are bounded by the coordinate planes, and the planes $x = 2,\ y = 3,\ z = 4$.

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10. (1 pt)

Find the outward flux of the vector field $\mathbf{F} = (x^3, y^3, z^2)$ across the surface of the region that is enclosed by the circular cylinder $x^2+y^2 = 25$ and the planes $z = 0$ and $z = 3$.

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11. (1 pt)

Evaluate $\int\int_M \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = (3xy^2, 3x^2y, z^3)$ and M is the surface of the sphere of radius 3 centered at the origin.

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12. (1 pt)

Let $\mathbf{F} = (y^2+z^3, x^3+z^2, xz)$. Evaluate $\int\int_{\partial W}\mathbf{F}\cdot d\mathbf{S}$ for each of the following regions W:

A. $x^2+y^2 \le z \le 10$ ____________

B. $x^2+y^2 \le z \le 10,\ x \ge 0$ ____________

C. $x^2+y^2 \le z \le 10,\ x \le 0$ ____________